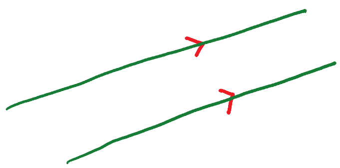
2.1 & 2.2 Angles and Parallel Lines - Key

Angles & Parallel Lines [2.1 & 2.2]

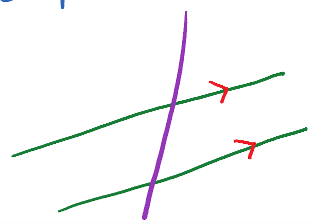
Parallel Lines

Two lines on the same plane that will never meet.



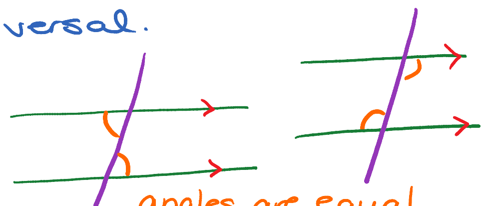
Transversal

a 3rd line that intersects two parallel lines



Alternate Interior Angles

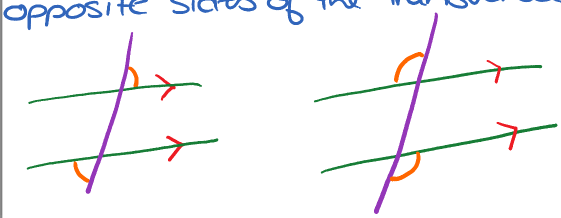
Two non-adjacent interior angles on opposite sides of a transversal.



angles are equal

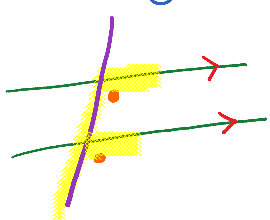
Alternate Exterior Angles

Two exterior angles formed between two lines and a transveral, on opposite sides of the transveral.



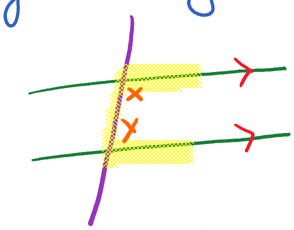
Corresponding Angles

Angles forming an "F" pattern



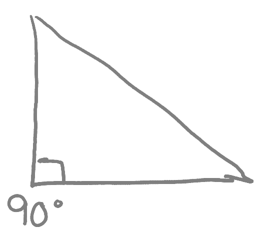
Interior Angles

Angles forming a "C" pattern.



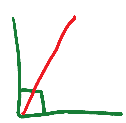
$x + y = 180°$

Right Triangle

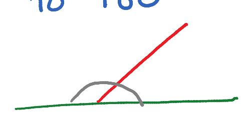


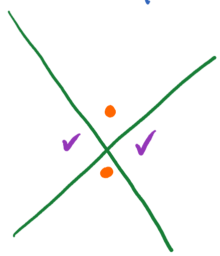
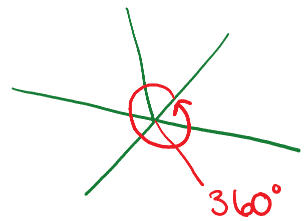
Complimentary & Supplementary Angles

angles add to 90°



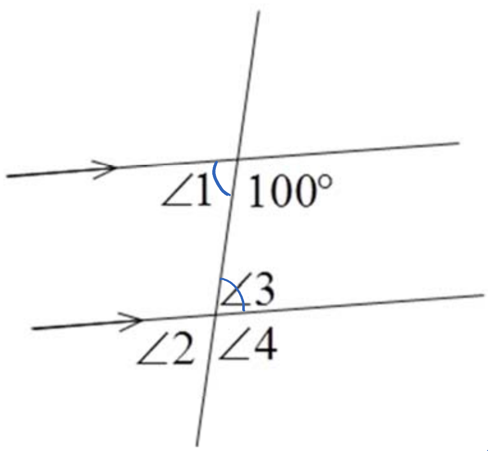
angles add to 180°



Vertically Opposite Angles	**Angles at a Point**
are equal 	add to 360° 

<u>So what does this look like when we apply it</u>:

Example 1: Try this.



∠1 = 80° angles on a line

∠2 = 80° Corresponding Angles

∠3 = 80° Alternate Interior Angles

∠4 = 100° Supplementary Angles

∠1 and ∠3 are equal

Conjecture

The conclusion, generalization, or educated guess which is arrived at by inductive reasoning is called a **conjecture**.

Conjectures **may** <u>or</u> **may not** be true.

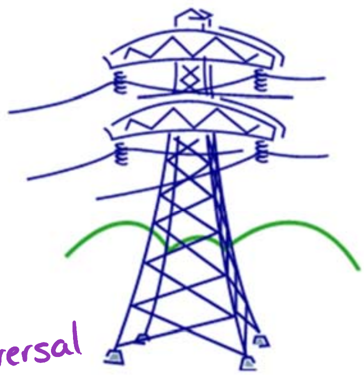
Converse

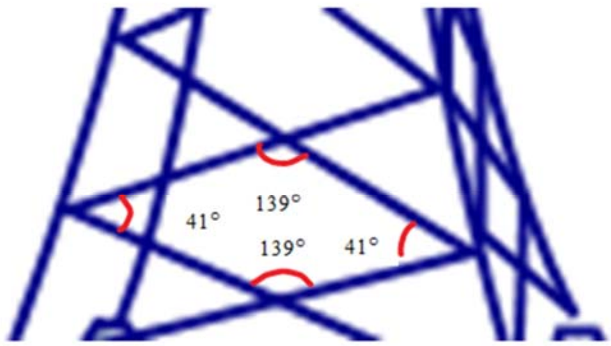
A conditional statement has a **converse** which may or may not be true. This occurs when the hypothesis and the conclusion are interchanged.

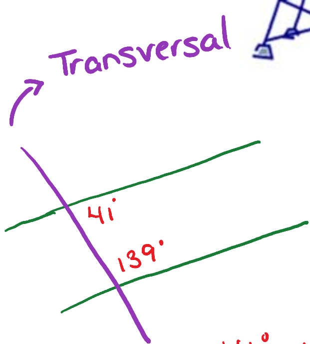
eg. **If** a Canadian citizen is able to vote, **then** the person is 18 years of age or older.

eg. **If** a person was born in Alberta, **then** the person must live in Calgary.

Example 2: An electrical tower is supported by braces. Prove that the braces are parallel.

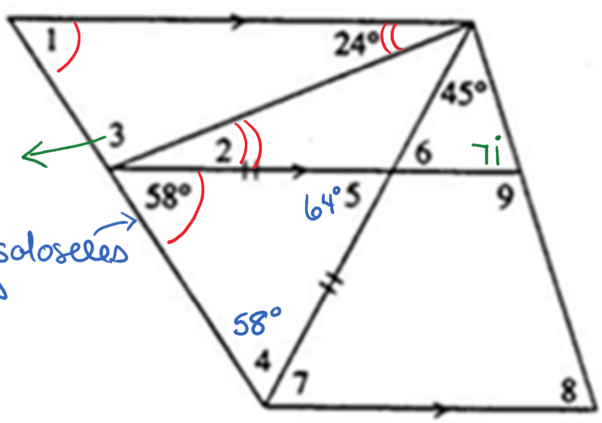






$41° + 139° = 180°$ – Interior Angles of Parallel lines

And one to try ☺. Find all missing numbered angles.



180°
- 24°
- 58°
98°

Isoloselles Δ

∠ 4 = 58° > Iso Δ

∠ 5 = 64° > ∠s in a Δ = 180°

∠ 6 = 64° > Vertically Opposite

∠ 7 = 64° > Alternate Interior

∠ 8 = 71° > Corresponding Angles

∠ 9 = 109° > Supplementary Angles

∠ 1 = 58° > Corresponding ∠s

∠ 2 = 24° > Alternate Interior

∠ 3 = 98° > Angles on a line

Vocabulary Intro

I'm going to read you 15 words. As I do, I want you to either sketch, explain or ask questions about each word as I read it. I will read the list twice. Don't worry about not knowing what the words mean yet. This is meant to help you to start "thinking math" again.

Which one do you think you'll do?

Here are the words: (First time just read – no visual, 2nd time- read only or display and read)

Parallel

Angle

Transversal

Corresponding angles

Converse

Interior Angles

Exterior Angles

Conjecture

Acute Angle

Right Angle

Obtuse Angle

Vertically Opposite Angles

Complementary Angles

Supplementary Angles

Symbols

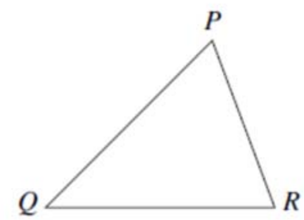
2.3 Angle Properties of Triangles - Key

Angle Properties of Triangles [2.3]

Gail was asked to prove the following

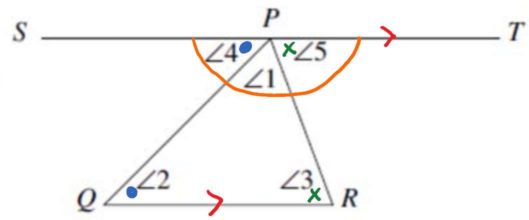
"In ΔPQR prove that $\angle P + \angle Q + \angle R = 180°$."

She has started the proof using a two column approach. Complete her proof.



Gail's Solution

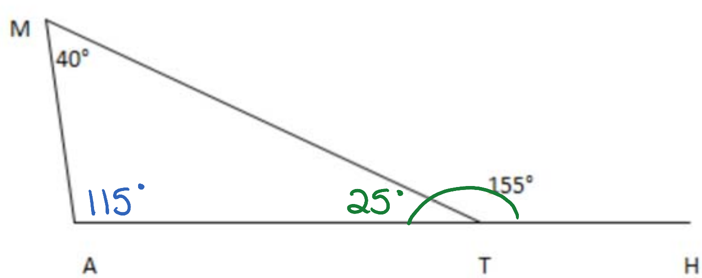
Draw a triangle PQR and a line segment ST through P parallel to QR. Give each angle a number as indicated in the diagram.



To prove: $\angle 1 + \angle 2 + \angle 3 = 180°$

Statement	Reason
$\angle 2 = \angle 4$	alternate interior ∠s
$\angle 3 = \angle 5$	alternate interior ∠s
$\angle 4 + \angle 1 + \angle 5$	straight line $= 180°$
$\therefore \angle 1 + \angle 2 + \angle 3 = 180°$	Addition Property

Example 1: Determine the measures of the unknown angles in Δ MAT.



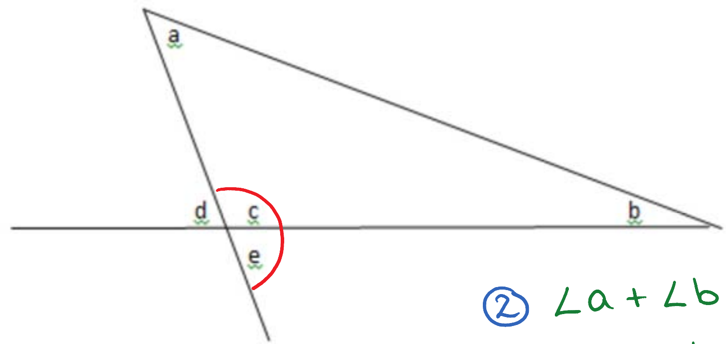
$\angle MTA = 180° - 155°$
$= 25°$

$\Delta MAT = 180°$
$180° - 40° - 25° = 115°$
$\therefore \angle MAT = 115°$

Notice anything else kinda interesting? (Think MA...)

Example 2: Prove ∠ e = ∠ a + ∠ b

Want ∠e alone



① $\angle e + \angle c = 180^\circ$

$\angle e = 180^\circ - \angle c$

Can we make this look like

② $\angle a + \angle b + \angle c = 180^\circ$

$\angle a + \angle b = 180^\circ - \angle c$

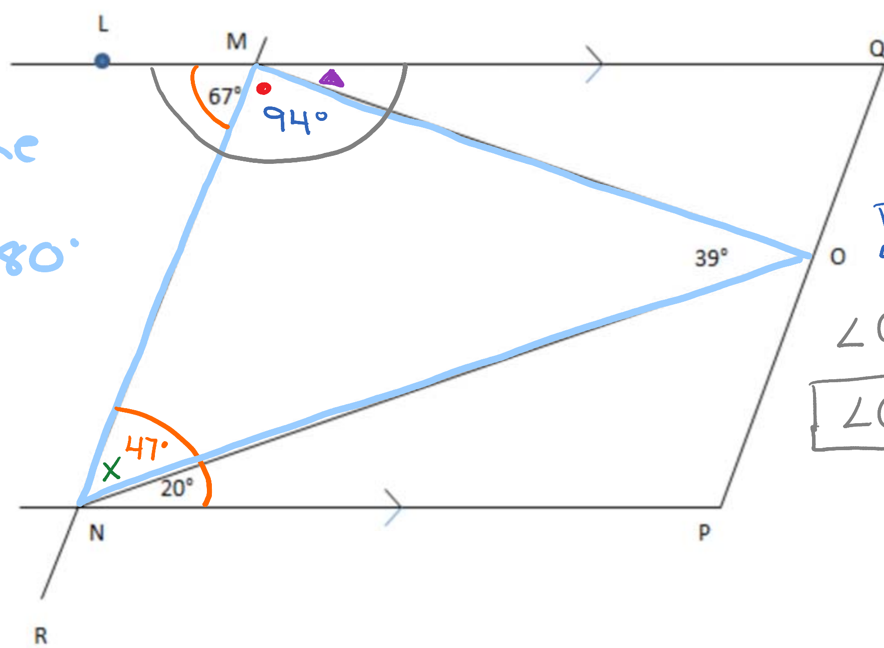
$\boxed{\therefore \angle e = \angle a + \angle b}$

Exterior Angles: Any exterior angle of a triangle is equal to the sum of the measures of the non-adjacent interior angles.

Example 3: Determine the measures of ∠ NMO, ∠MNO, and ∠QMO.

See the Δ?

Δs = 180°



$\angle MNO + 20^\circ = 67^\circ$

$\boxed{\angle MNO = 47^\circ}$

$\angle NMO = 180^\circ - 39^\circ - 47^\circ$

$\boxed{\angle NMO = 94^\circ}$

$\angle QMO = 180^\circ - 67^\circ - 94^\circ$

$\boxed{\angle QMO = 19^\circ}$

2.4 Angle Properties in Polygons - Key

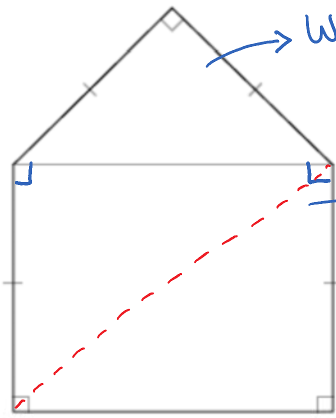
Angle Properties in Polygons [2.4]

adding

Explore: A pentagon has three right angles and four sides of equal length. What is the sum of the measures of the angles in the pentagon?

5 sides

We know $\Delta s = 180°$



Pentagon

$$\begin{array}{r} 180° \\ +\,360° \\ \hline \underline{\underline{540°}} \end{array}$$

Two options

Rectangles have four 90° angles (=360°)

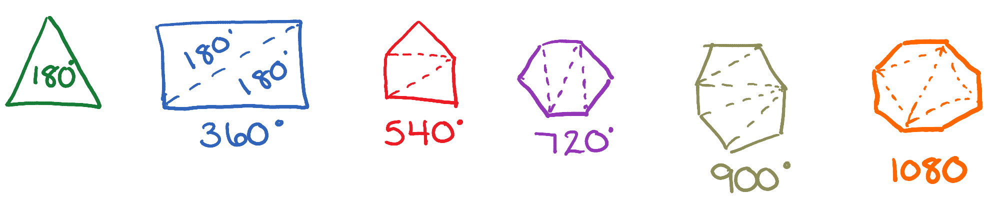
OR

Create two triangles, each with 180° (=360°)

Draw the polygons listed in the table below. Create triangles to help you determine the sum of the measures of their interior angles.

We are looking for a pattern or relationship between # of sides / # of Δ / and their Sum.

Polygon	Number of Sides	Number of Triangles	Sum of Angle Measures
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°



Here's what I notice:

The # of Δs was always 2 less than the # of sides.

Sum (+)

$S(n) = 180°(n-2)$

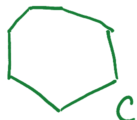
times # of Δs inside the polygon

# of sides

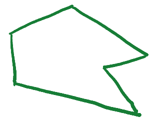
Angles per Δ

Now we know how to find sum, what about each interior angle?

Convex Polygon: A polygon with each interior angle less than 180°



convex



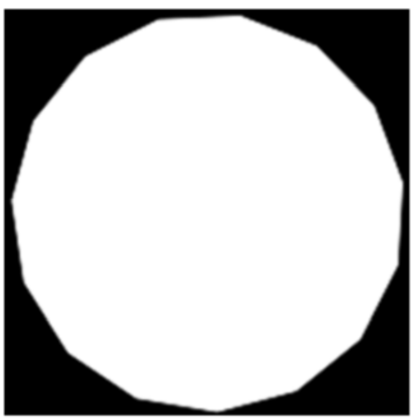
non-convex (concave)

* If I know the sum, then I can divide the sum by the # of sides to find the interior angles! *

↳ What about exterior angles? Check out page 95

Example 1: Determine the measure of each interior angle of a regular 15-sided polygon (a pentadecagon).

n



*Don't want to draw Δs? Use the formula:

$S(n) = 180°(n-2)$

$S(15) = 180°(15-2)$

$S(15) = 180°(13)$

$S(15) = 2340°$

↳ Sum

$\frac{2340°}{15} = 156°$

↓

each interior angle

n= 6

Example 2: Can a tiling pattern be created using regular hexagons and equilateral triangles that have the same side length. How can you tell? (Psssst... read through example 3 on page 98, first)

① hexagon

$S(n) = 180°(n-2)$

$S(6) = 180°(6-2)$

$S(6) = 180°(4)$

$S(6) = 720°$ (total)

↓

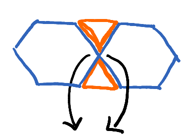
$\frac{720°}{6} = 120°$ (each)

② Equilateral Δ

Equal sides AND Equal angles

$\therefore \frac{180°}{3} = 60°$

③ If we put 2 hexagons together



* Angles at a point=360°

$120° + 60° + 120° + 60° = 360°$

This means the pattern would work! (No one wants holes in their floor)